Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 80 students entered a dancing competition.
The table gives information about the length of time, in minutes, for which each student spent dancing.

Frequency	**Time (m)**
$0 < m \leq 12$	11
$12 < m \leq 24$	25
$24 < m \leq 36$	23
$36 < m \leq 48$	15
$48 < m \leq 60$	6

Work out an estimate for the mean length of time the students spent dancing.

$6 \times 11 = 66$ +

$18 \times 25 = 450$ +

$30 \times 23 = 690$ +

$42 \times 15 = 630$ +

$54 \times 6 = 324$

2160

$2160 \div 80$

27 minutes

(Total for Question 1 is 4 marks)

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2 Solve $3(2 - 4x) = 5 - 8x$
Show clear algebraic working.

$6 - 12x = 5 - 8x$

$6 - 5 = -8x + 12x$

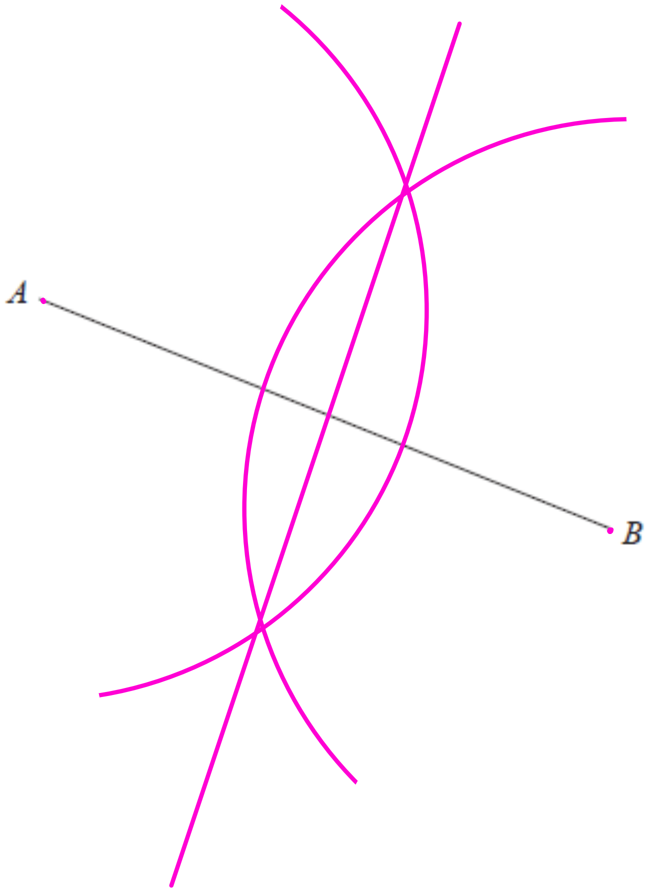
$1 = 4x$

$x = \frac{1}{4}$

$x =$ 1/4

(Total for Question 2 is 3 marks)

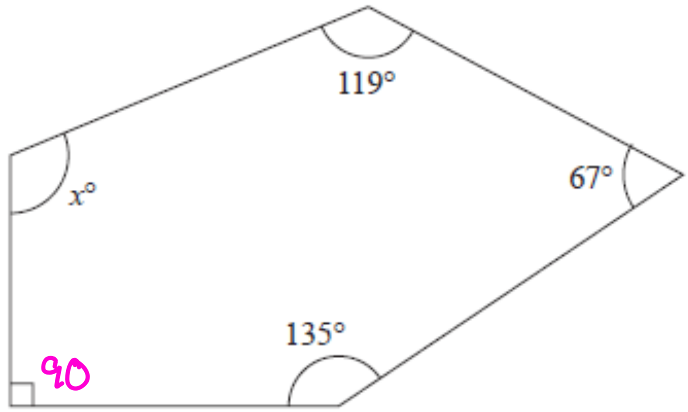
3 Use ruler and compasses only to construct the perpendicular bisector of line AB
You must show all your construction lines.



(Total for Question 3 is 2 marks)

4 The diagram shows a pentagon.

Work out the value of x



$540 - (119 + 67 + 135 + 90)$

$= 540 - 411$

$x = $ ....129....

(Total for Question 4 is 3 marks)

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5 In a box, there are only green sweets, orange sweets and yellow sweets.

There are 280 sweets in the box so that

the number of green sweets : the number of orange sweets = 2 : 3
and
the number of orange sweets : the number of yellow sweets = 1 : 5

Work out how many green sweets there are in the box.

G O
2 3

O Y
1 5
×3 3 15 ×3

80

$\underbrace{\text{G : O : Y} \quad 2 \quad 3 \quad 15}$

$280 \div 20 = 14$

2×14
$= 28$

....28....

(Total for Question 5 is 3 marks)

6 Shane bought a car. The amount Shane paid for the car was $32 000

Theresa also bought a car. To pay for this car, Theresa paid a deposit of $18 000 together with 14 monthly payments of $1160

Theresa paid more for her car than Shane paid for his car.

(*a*) Work out how much more Theresa paid as a percentage of the amount Shane paid.

Shane
32000

Theresa
18000 + 14 × 1160
= 34240

Difference = 34240 - 32000 = 2240

% = $\frac{2240}{32000} \times 100 = 7$

.......7.......%**(4)**

Kylie bought a van. After 1 year, the value of the van was $39 865
During this year, the value of the van decreased by 15%

(*b*) Work out the value of the van when Kylie bought it.

95% = 39865
÷95
1% = 469
×100
100% = 46900

$.......46900....... **(3)**

(Total for Question 6 is 7 marks)

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7 Some members of a library were asked to name the type of book that they each liked to read the best.

One of the members is chosen at random. The table shows information about the probability of the type of book that this member answered.

Type of book	comedy	romance	mystery	thriller
Probability	0.24	0.40	0.27 3*x*	0.09 *x*

48

48 members answered comedy books.

Work out how many of the members answered mystery books.

1 - (0.24 + 0.4)
= 0.36

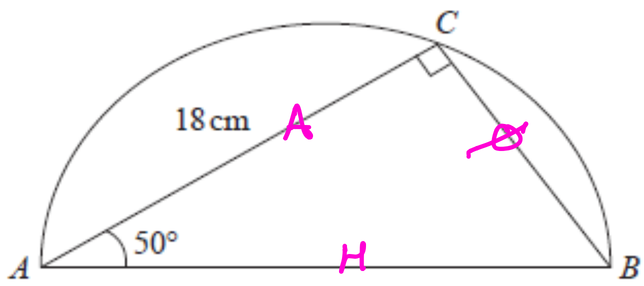
0.36 ÷ 4 = 0.09

0.24 = 48
÷24 ÷24
0.01 = 2
×9 ×9
0.09 = 18

so 0.27 = 18×3 ....54.......

(Total for Question 7 is 4 marks)

8 The diagram shows a triangle ABC inside a semicircle.



A, B and C are points on the semicircle.

AB is the diameter of the semicircle.

Angle $ACB = 90°$ Angle $BAC = 50°$ $AC = 18$ cm

Work out the perimeter of the semicircle.
Give your answer correct to 2 significant figures.

$\cos 50 = \frac{18}{AB}$ $AB = \frac{18}{\cos 50}$ $AB =$ diameter

$= 28.00\ldots$

Perimeter $= \pi \times AB \times \frac{1}{2} + AB$

$= \frac{1}{2}\pi \times 28.00\ldots + 28.00\ldots$

$= 71.99\ldots$

72 cm

(Total for Question 8 is 5 marks)

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9 (*a*) Write 6.25×10^{-4} as an ordinary number.

0.000625 **(1)**

(*b*) Work out $(2.4 \times 10^{12}) \div (9.6 \times 10^{4})$ Give your answer in standard form.

2.5×10^{7} **(2)**

(Total for Question 9 is 3 marks)

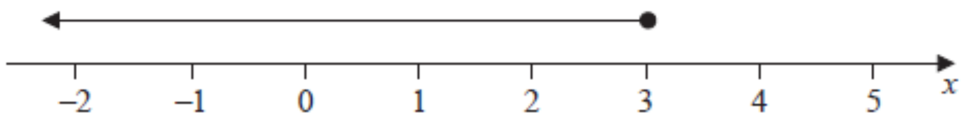
10 (*a*) Factorise $y^2 - 2y - 48$

$6 \times 8 = 48$

$6 - 8 = -2$

$(x+6)(x-8)$ .... **(2)**

(*b*) Write down the inequality shown on the number line



$x \leq 3$ .... **(1)**

(*c*) Solve the inequality $7w + 6 > 12w + 14$

$-8 > 5w$

$w < -\frac{8}{5}$

$w < -8/5$ .... **(3)**

(Total for Question 10 is 6 marks)

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11 The region **R**, shown shaded in the diagram, is bounded by the straight lines with equations

$2x + y = 6$

$2y = 5x + 1$

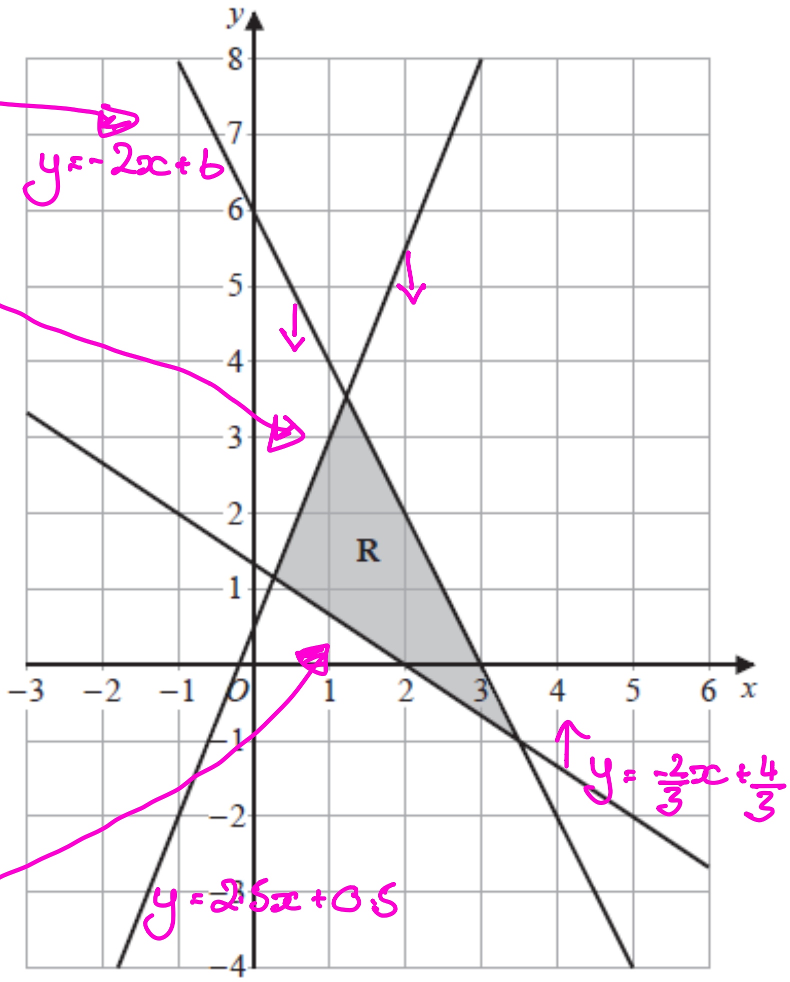
$3y + 2x = 4$

Write down the three inequalities that define **R**

$2y + x \leq 6$ ....

$2y \leq 5x + 1$ ....

$3y + 2x \geq 4$ ....



(Total for Question 11 is 3 marks)

12 $3^{\frac{1}{2}} \times 3^{\frac{2}{5}} = 3^m$

(*a*) Work out the value of m

$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

$m =$ ....9/10.... **(1)**

$5^{-10} \div 5^{-4} = 5n$

(*b*) Work out the value of n

$-10 - -4 = -10 + 4 = -6$

$n =$ ....-6.... **(1)**

(Total for Question 12 is 2 marks)

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13 Expand and simplify $3x(2x - 5)^2$ Show clear algebraic working.

$3x(2x - 5)(2x - 5)$

$= 3x(4x^2 - 20x + 25)$

$= 12x^3 - 60x^2 + 75x$

....$12x^3 - 60x^2 + 75x$....

(Total for Question 13 is 3 marks)

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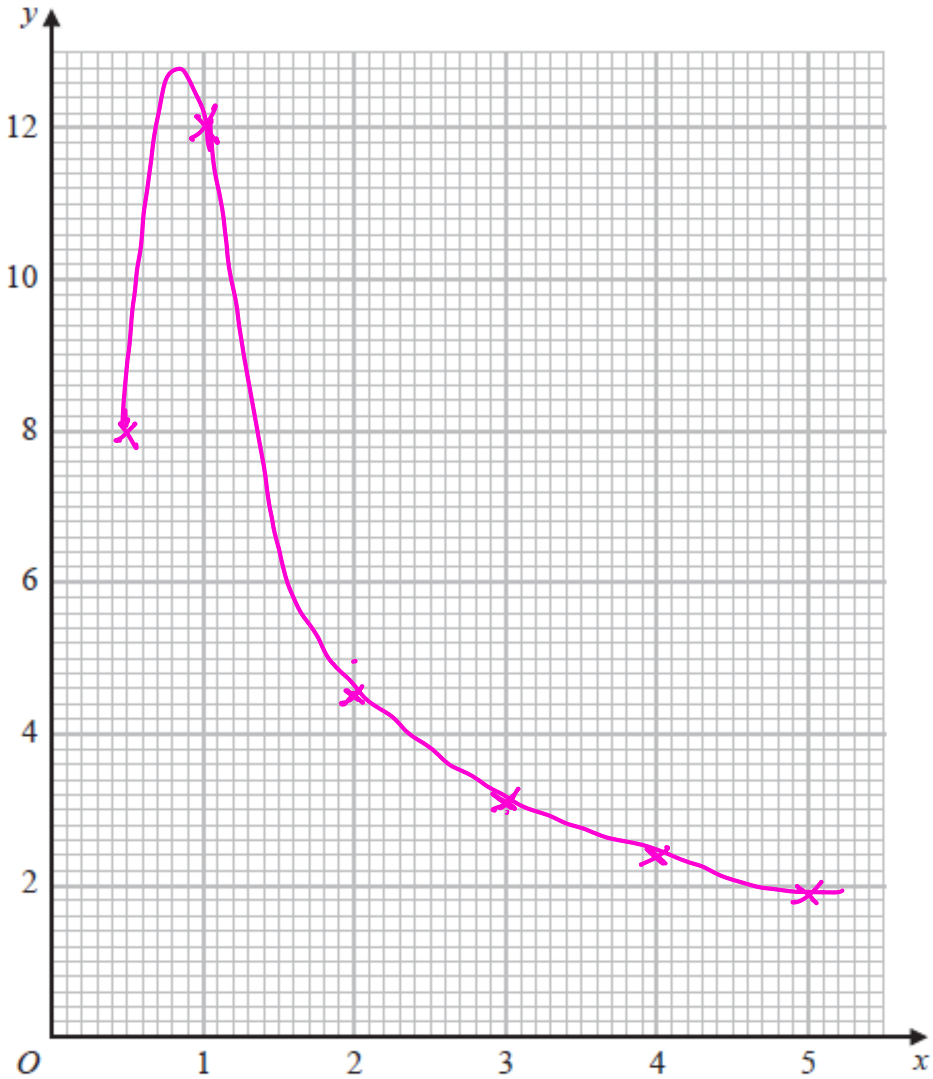
14 (*a*) Complete the table of values for $y = \frac{2}{x}\left(5 - \frac{1}{x}\right)$

x	0.5	1	2	3	4	5
y	8	12	4.5	3.1	2.4	1.9

(1)

(*b*) On the grid, draw the graph of $y = \frac{2}{x}\left(5 - \frac{1}{x}\right)$ for $0.5 \leq x \leq 5$



(2)

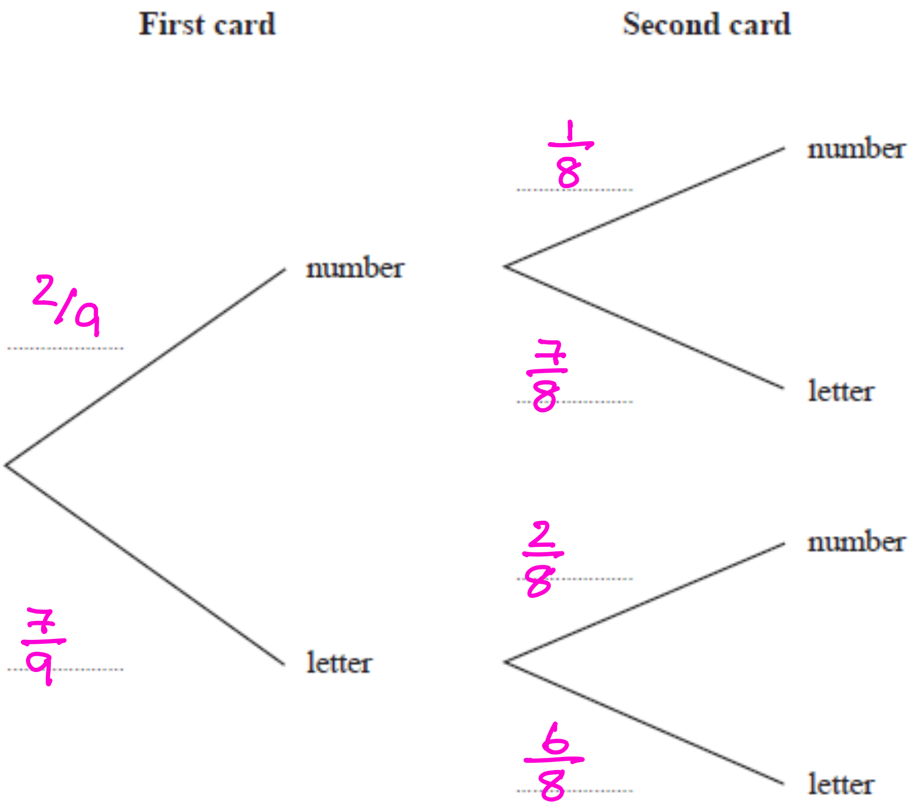
(Total for Question 14 is 3 marks)

15 Here are 9 cards. Each card has either a number on it or a letter on it.

A	7	H	F	E	D	G	5	C

Tomas is playing a game. Tomas takes at random one of the cards and keeps it. Tomas then takes at random another card and keeps it. ← LOOK

(*a*) Complete the probability tree diagram.



(2)

(*b*) Work out the probability that each of the two cards has a number on it.

$\frac{2}{9} \times \frac{1}{8}$

$\frac{2}{72}$ OR $\frac{1}{36}$ **(2)**

(*c*) Work out the probability that there will be one card with a number on it and one card with a letter on it.

$\frac{2}{9} \times \frac{7}{8} + \frac{7}{9} \times \frac{2}{8}$

$= \frac{14}{72} + \frac{14}{72}$

$28/72$ OR $\frac{7}{18}$ **(3)**

(Total for Question 15 is 7 marks)

16 Here is a shape formed from two triangles *ABC* and *CDE*. *ACD* and *BCE* are straight lines.

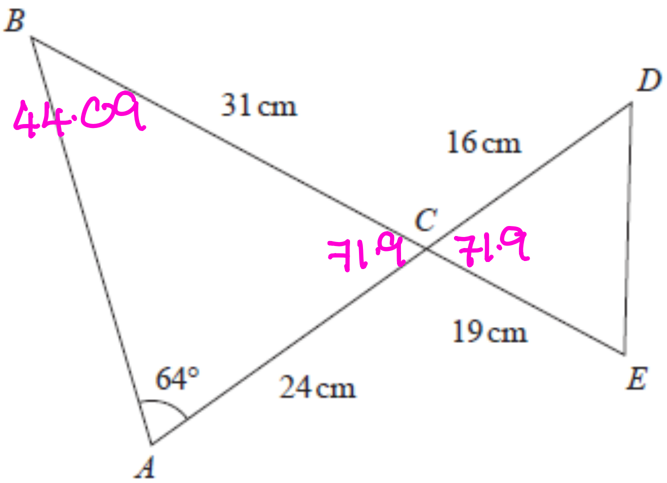


Diagram NOT accurately drawn

$AC = 24$ cm $\quad BC = 31$ cm $\quad CE = 19$ cm $\quad CD = 16$ cm $\quad$ Angle $BAC = 64°$

Work out the length of *DE*. Give your answer correct to 3 significant figures.

$$\frac{31}{\sin 64} = \frac{24}{\sin ABC} \qquad \sin ABC = \frac{24 \times \sin 64}{31}$$

$$ABC = 44.09\ldots$$

$$ACB = 71.90\ldots$$

$$DE^2 = 19^2 + 16^2 - 2 \times 19 \times 16 \times \cos 71.9$$

$$DE = \sqrt{428.166\ldots} = 20.69\ldots$$

20.7 .... cm

(Total for Question 16 is 5 marks)

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17 y is inversely proportional to $\sqrt{x}$

$y = c^4$ when $x = c^2$ where c is a positive constant.

Find a formula for y in terms of x and c
Give your answer in its simplest form.

$$y \propto \frac{1}{\sqrt{x}} \qquad y = \frac{k}{\sqrt{x}} \qquad y = c^4 = \frac{k}{\sqrt{c^2}} \qquad \therefore k = c^4 \times c = c^5$$

$$\text{so } y = \frac{c^5}{\sqrt{x}}$$

$y = \dfrac{c^5}{\sqrt{x}}$ ....

(Total for Question 17 is 3 marks)

18 The function f is such that $f(x) = \dfrac{k}{x}$ where $x \neq 0$ and k is an integer.

(*a*) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$y = \dfrac{k}{x}$ $\quad$ $x = \dfrac{k}{y}$

$f^{-1}(x) = $ ....$\dfrac{k}{x}$........ **(1)**

The function g is such that $g(x) = 2 - 3x^4$ where $x \neq 0$

The function h is such that $h(x) = \dfrac{3x}{2-x}$ where $x \neq 2$

(*b*) (i) Find g(−2)

$2 - 3(-2)^4 = 2 - 3 \times 16$

........-46........ **(1)**

(ii) Express the composite function hg in the form $hg(x) = ...$
Give your answer in its simplest form.

$hg(x) = \dfrac{3(2-3x^4)}{2-(2-3x^4)} = \dfrac{6-9x^4}{2-2+3x^4}$

$= \dfrac{\cancel{6}^{2} - \cancel{9}^{3}x^4}{\cancel{3}x^4}$

$hg(x) = $ ....$\dfrac{2-3x^4}{x^4}$........ **(2)**

(Total for Question 18 is 4 marks)

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19 The acceleration, a, of an object is given by

$$a = \frac{v-u}{t}$$

where

$v = 45.23$ correct to 2 decimal places
$u = 5.12$ correct to 2 decimal places
$t = 8.5$ correct to 2 significant figures

LB	UB
45.225	45.235
5.115	5.125
8.45	8.55

By considering bounds, work out the value of a to a suitable degree of accuracy.
Show your working clearly and give a reason for your answer.

$a_{UB} = \dfrac{45.235 - 5.115}{8.45} = 4.74792..$

$a_{LB} = \dfrac{45.225 - 5.125}{8.55} = 4.6900..$

Both round to 4.7 to 1 dp

$a = $ ....4.7........

(Total for Question 19 is 5 marks)

20 The radius of a right circular cylinder is x cm.

The height of the cylinder is $\left(\frac{800}{\pi x} - x\right)$ cm.

The volume of the cylinder is V cm^3

Find the maximum value of V Give your answer correct to the nearest whole number.

Volume : $\pi x^2 \left(\frac{800}{\pi x} - x\right) = 800x - \pi x^3$

$\frac{dV}{dx} = 800 - 3\pi x^2$ $\quad 800 - 3\pi x^2 = 0$

$\frac{800}{3\pi} = x^2$

$x = \sqrt{\frac{800}{3\pi}} = 9.21\ldots$

$V_{MAX} = 800 \times 9.21.. - \pi \times 9.21.^3$

$= 4913.69..$

4914

(Total for Question 20 is 5 marks)

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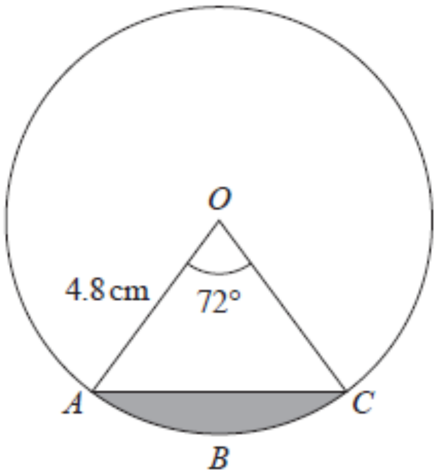
21 The diagram shows the cross section of a circular water pipe.



OABC is a sector of the circle, centre *O*

The shaded region in the diagram represents the water flowing in the pipe.

The water flows at 14 cm/s in the pipe.

Work out the volume of water that has flowed through the pipe in 3 minutes.
Give your answer in cm^3, correct to 3 significant figures.

sector $\frac{72}{360} \times \pi \times 4.8^2 = 14.476\ldots$

triangle $\frac{1}{2} \times 4.8 \times 4.8 \times \sin 72 = 10.956\ldots$

segment $= 14.476.. - 10.956 = 3.520$ cm^2

$3.520 \times 14 \times 3 \times 60$

8870 cm^3

(Total for Question 21 is 5 marks)

22 The first term of an arithmetic series is $(2t + 1)$ where $t > 0$
The nth term of this arithmetic series is $(14t - 5)$

The common difference of the series is 3

The sum of the first n terms of the series can be written as $p(qt - 1)^r$ where p, q and r are integers.

Find the value of p, the value of q and the value of r
Show clear algebraic working.

n_{th} $\quad a + (n-1)d$

$14t - 5 = 2t + 1 + (n-1)3$

$12t - 6 = 3n - 3 \quad 12t - 3 = 3n \quad$ so $n = 4t - 1$

$S_n = \frac{4t-1}{2}[2t + 1 + 14t - 5] = \frac{4t-1}{2}(16t - 4)$

$= \left(\frac{4t-1}{\cancel{2}}\right) \times \cancel{4}^{2}(4t - 1) = 2(4t-1)^2$

$p =$ 2 $\quad q =$ 4 $\quad r =$ 2

(Total for Question 22 is 4 marks)

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23 $ABCD$ is a kite. $\quad AB = AD$ and $CB = CD$

The point B has coordinates $(k, 1)$ where k is a negative constant.
The point D has coordinates $(8, 7)$

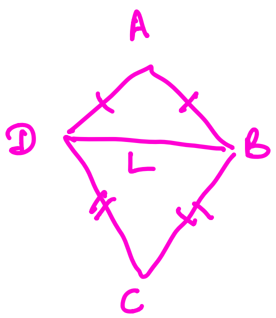
The straight line **L** passes through the points B and D

The straight line **L** is parallel to the line with equation $5y - 3x = 6$

Find an equation of AC. Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show your working clearly.

$5y = 3x + 6$

$y = \frac{3}{5}x + \frac{6}{5}$



so line L has a gradient of $\frac{3}{5}$

so L $\Rightarrow$ $y = \frac{3}{5}x + c$

$(8, 7)$
$\ x \ \ y$

$7 = \frac{3}{5} \times 8 + c \quad c = 2.2$

L $\Rightarrow$ $y = \frac{3}{5}x + \frac{11}{5}$

$(B) \ \left(1 - \frac{11}{5}\right) \times \frac{5}{3} = y \quad k = -2 \quad B = (-2, 1)$

midpoint $DB = (3, 4)$

$AC \Rightarrow y = -\frac{5}{3}x + c \quad 4 = -\frac{5}{3} \times 3 + c \quad c = 9$

$y = -\frac{5}{3}x + 9 \quad 3y = -5x + 27 \quad 5x + 3y = 27$

(Total for Question 23 is 6 marks)

24 *OAED* is a quadrilateral.

$AE = a + 9b$

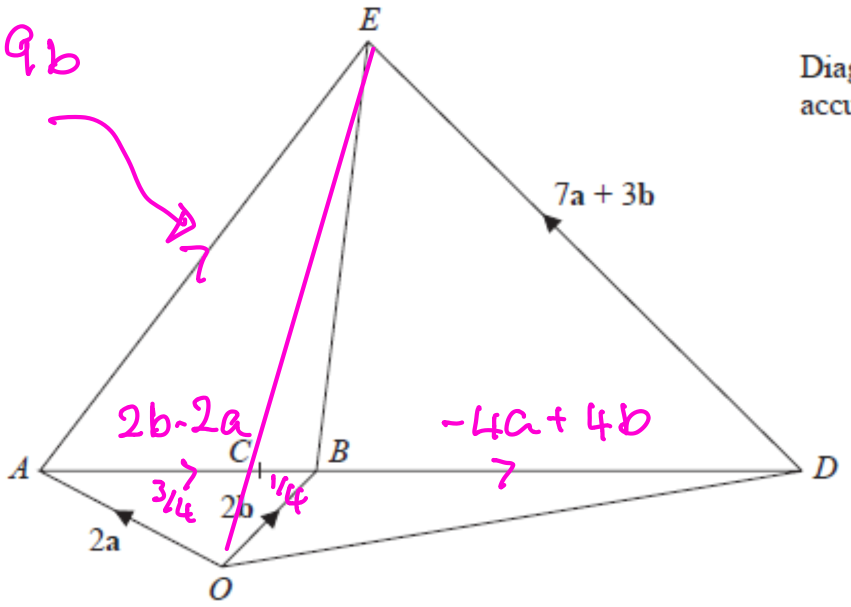


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = 2\mathbf{b}$ $\overrightarrow{DE} = 7\mathbf{a} + 3\mathbf{b}$

$\overrightarrow{AB} = -2a + 2b$

$AB : BD = 1 : 2$

$BD = -4a + 4b$

The point *C* on *AB* is such that *OCE* is a straight line.

Use a vector method to find the ratio of *OC* : *CE*

$\overrightarrow{OE} = 2b - 4a + 4b + 7a + 3b = 3a + 9b$

$\overrightarrow{OC} = 2a + \lambda(-2a + 2b)$

$= (2 - 2\lambda)a + 2\lambda b$

$\begin{pmatrix} a \\ b \end{pmatrix}$ $\dfrac{2 - 2\lambda}{2\lambda} = \dfrac{3}{9}$ $18 - 18\lambda = 6\lambda$

$24\lambda = 18$ $\lambda = \dfrac{3}{4}$

$OC = (2 - 1.5)a + 1.5b = 0.5a + 1.5b$

$CE = -1.5b + 1.5a + a + 9b = 2.5a + 7.5b$

so $OC : CE = 1 : 5$

1 : 5

(Total for Question 24 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS